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Application of effectiveness-NTU relationship to parallel flow microchannel heat exchangers subjected to external heat transfer

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ABSTRACT

In this paper the thermal performance of parallel flow microchannel heat exchangers subjected to constant external heat transfer has been theoretically analyzed. Equations for predicting the axial temperatures as well as the effectiveness of the fluids of a microchannel heat exchanger operating under laminar flow conditions have been developed. In addition, an equation for determining the heat transfer between the fluids has also been formulated. Irrespective of the heat capacity ratio, for a specific NTU, external heating always decreases and increases the effectiveness of the hot and cold fluid, respectively. The opposite trend in the effectiveness of the fluids is observed when they are subjected to external cooling. Moreover, under unbalanced flow conditions (heat capacities of two fluids are not equal) the effectiveness of the fluids depended on the fluid with the lowest heat capacity. Among the two unbalanced flow conditions (heat capacities of the two fluids are not equal) the effectiveness of the fluids is greatest when the hot fluid has the lowest heat capacity. At a given NTU, reduction in heat capacity ratio improved the effectiveness of the fluids. Under certain operating conditions temperature cross over was observed in the heat exchanger.

1. Introduction

Microchannel heat exchangers (MCHXs) can be broadly classified as fluidic devices that employ channels of hydraulic diameter smaller than 1 mm. Mehendale et al. [1] have provided a more specialized classification of heat exchangers with channels having hydraulic diameter below 1 mm. Mehendale et al. [1] termed heat exchangers with channels of hydraulic diameter between 1 μm and 100 μm as micro heat exchanger and those with channels of hydraulic diameter between 100 μm and 1 mm as meso heat exchangers. In addition, those heat exchangers employing channels with hydraulic diameter between 1 mm and 6 mm and those with channels greater than 6 mm are termed as compact and conventional heat exchangers, respectively. However, heat exchangers channels with hydraulic diameter smaller than 1 mm would be addressed as microchannel heat exchangers (MCHXs) in this article. Just like convectional heat exchangers these devices are also used for transferring heat between two fluids, which are at different temperatures. MCHXs offer several advantages over conventional heat exchangers, some of which are 1) compact size, 2) high surface area density, and 3) enhanced heat transfer coefficients. Present day MCHXs are made using materials such as silicon, aluminum, stainless steel, glass, and ceramics. Selection of fabrication material depends on the temperature and pressure range of operation. MCHXs are mostly used as recuperators in microreactors, microchannel fuel cells and microminiature refrigerators. MCHXs have been put to new uses recently. One of the latest applications of MCHXs has been in the cooling of cells [2]. A major issue associated with employing microchannels in heat exchangers is pressure drop across the microchannels. Pressure drop is inversely proportional to the hydraulic diameter of the channels. Thus, with reduction in hydraulic diameter the pressure drop increases for a particular flow rate and channel length. Nevertheless due to the above mentioned advantages MCHXs have been commercialized by firms like Chart Industries® and Heatric®.

In recent years there has been a thrust towards integrating two or more unit operations in a MCHX [3,4]. A chemical reactor integrated in a MCHX is one such device [3]. In this device chemical reactions are carried out in one set of channels and a cooling/heating fluid is passed through the other. The presence of heating/cooling fluid helps to control the temperature of the reactants. Thus, this device may have better product yield, reduced processing time and require less infrastructure. Improvement in product yield is achieved by better control of reactants temperatures. Reduction

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Nomenclature

A	heat transfer surface area (m^2)
C	heat capacity of individual fluid ($W\,K^{-1}$)
C_R	ratio of minimum heat capacity to the heat capacity of the individual fluid, $C_R = C_{min}/C$
C_r	ratio of minimum heat capacity to the maximum heat capacity, $C_r = C_{min}/C_{max}$
K_1,K_2	terms used in equation (5) and (6)
Kn	Knudsen number
L	length of the MCHX (m)
MCHX	microchannel heat exchanger
Ma	Mach number
NTU	number of transfer units, NTU $= UA/C_{min}$
q	heat transfer (W)
q''	heat flux ($W\,m^{-2}$)
Q	nondimensional external heat transfer parameter, $Q = q_{ext}/q_{max} = q''A/C_{min}(T_{h,i} - T_{c,i})$
Q^*	nondimensional heat transfer parameter, $Q^* = q/q_{max} = q/C_{min}(T_{h,i} - T_{c,i})$
Re	Reynolds Number
T	temperature of the fluid (K)
ΔT	temperature difference (K)
U	overall heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
x	axial distance (m)
Z	nondimensional axial distance, $Z = x/L$

Greek symbols

ε	effectiveness, $Q = q_{act}/q_{max} = q_{act}/C_{min}(T_{h,i} - T_{c,i})$
θ	nondimensional fluid temperature, $\theta = T - T_{c,i}/T_{h,i} - T_{c,i}$

Subscript

act	actual
c	cold fluid
cr	cross over
ext	external
h	hot fluid
i	inlet
lm	log mean
min	minimum
max	maximum
net	net
o	outlet
PF	parallel flow

in processing time is obtained as both heat transfer and chemical reactions are carried out simultaneously in a single device. The infrastructure needed for constructing a MCHX and a reactor separately is obviously reduced by integrating both into one single device. Delsman et al. [4] recently developed a microscale portable fuel processor which comprised of three microdevices of which one was a chemical reactor integrated in a counter flow heat exchanger. In this device one set of channels were used for performing chemical reactions, preferential oxidation of carbon monoxide, and a coolant was circulated through the other set of channels. Using this device Delsman et al. [4] was able to improve the chemical reaction as well as salvage about 90% of the heat generated due to chemical reaction. Another device that can be developed by this approach, i.e. by combining unit operations such as heat and mass transfer is a micromixer integrated in a MCHX. This device too can help to reduce the processing time as well as the infrastructure. A MCHX fabricated with sinusoidal or serpentine channels would act as a mixer as well. Channels with these profiles have been already used as passive mixers [5].

Currently MCHXs are designed using the same ε-NTU relationships that were developed for designing conventional heat exchangers [6]. However, this may not be the proper approach since certain phenomena that are often insignificant in conventional heat exchangers may be significant in MCHXs. These phenomena include external heat transfer, viscous heating, axial heat conduction, entrance effects, and wall slip conditions. Of the above mentioned phenomena only external heat transfer is considered in this paper. External heat transfer refers to the situation where the fluids in a MCHX thermally interact with an external heat source which could be the ambient or chemical reaction chambers or combustors. External heat transfer usually exists due to the inability to properly thermally isolate a MCHX as compared to that provided for a conventional heat exchanger. It is hard to thermally insulate a MCHX by wrapping it in materials of low thermal conductivity or by placing it in a dewar as these techniques would prevent the integration of a MCHX on the same substrate with other microdevices. Depending on the external heat source the heat exchanger can be subjected to a constant/variable external temperature or heat flux [7]. In this paper it is assumed that the heat exchanger is subjected to a uniform/constant axial heat flux.

Barron [8] developed an analytical model for accessing the effect of external heat transfer between the ambient and the fluids on the hot fluid effectiveness of a counter flow heat exchanger. In this particular heat exchanger the fluids were subjected to a constant temperature. He developed two models, in each of these models only one of the fluids interacted thermally with the ambient. He provided equations for determining the axial temperature of the fluids in both models. From these models Barron [8] noticed that external heating always reduced the effectiveness of the hot fluid and in extreme cases it became negative. In his study the NTU was defined with respect to the fluid that was externally heated. Thus, for these models the conventional NTU will be same as the NTU defined in this paper only under balanced flow conditions or when the fluid subjected to external heat transfer has the lowest heat capacity.

Chowdhury and Sarangi [9] analyzed the effect of external heat transfer on the thermal performance of the fluids in a double pipe counter flow heat exchanger. External heat transfer occurred due to the difference in temperature of the fluids in the heat exchanger and the ambient. In their model it was assumed that the hot and cold fluids were being pumped through the inner and outer tube, respectively. Only the fluid in the outer tube was subjected to external heat transfer. Equations were developed for predicting the temperature of the fluids as well as the hot fluid effectiveness of this heat exchanger. They developed a novel concept called effective-NTU for assessing the effect of external heat transfer on the thermal performance of a counter flow heat exchanger. Effective-NTU is the NTU back calculated from the effectiveness of the heat exchanger subjected to external heat transfer. The conventional ε-NTU equations are used for determining effective-NTU. For a heat exchanger subjected to external heating, the effective-NTU is lower than the NTU for which it is designed, defined as design-NTU by Chowdhury and Sarangi [9], without considering external heat transfer. The difference in design-NTU and effective-NTU represents the loss of heat transfer surface area due to external heat transfer. For a heat exchanger free of external heat transfer the effective-NTU is same as the design-NTU. For a heat exchanger subjected to external cooling the effective-NTU would be greater than the design-NTU. They analyzed a heat exchanger in which the ambient temperature was same as the inlet temperature of the hot fluid and the cold fluid had the minimum heat capacity. For a particular heat capacity ratio (C_r) the effective-NTU decreased with reduction in thermal resistance between the ambient and outer fluid. On the other hand, decreases in the heat capacity ratio (C_r) reduced the effectiveness for a specific thermal resistance between the ambient and the outer fluid. They observed that at a specific thermal resistance and heat capacity ratio effective-NTU reached an asymptote value with respect to design-NTU. This asymptote value of NTU was referred to as the terminal-NTU by these authors [9]. Terminal-NTU increased with increase in thermal resistance for a specific heat capacity ratio. Moreover, terminal-NTU decreased with increases in heat capacity ratio for a specific thermal resistance between the ambient and the outer fluid. One of the primary limitations of this model is that it can be used only when the inner tube is occupied by the hot fluid and the outer tube is filled with the cold fluid.

Prasad [10] developed analytical equations for predicting the axial temperature of the fluids in a double pipe heat exchanger that thermally interacted with the ambient. In these heat exchangers only the fluid in the outer tube exchanged heat with the ambient. In addition, the ambient temperature was kept constant over the length of the heat exchanger. He developed models for both counter and parallel flow arrangements. For all the cases that were analyzed using this model, the hot fluid was pumped through the outer tube. The axial temperature of the hot and cold fluid for both flow arrangements was lower than that in a heat exchanger without external heat transfer whenever the ambient temperature was lower than the inlet temperature of the hot fluid. Moreover, according to this model any reduction in the thermal resistance between ambient and outer tube had a positive and negative impact on the hot and cold fluid, respectively. Similar effect on the temperature of the fluids was observed with reduction in the heat capacity ratio. In the equations developed by Prasad [10] NTU was defined with respect to the heat capacity of the fluid in the inner tube rather than with respect to the minimum heat capacity. Therefore, in order to improve the applicability of the model Prasad [10] has provided additional equations for relating the conventional NTU with the NTU defined in the paper.

Ameel and Hewavithrana [11] analyzed the effect of external heat transfer on the thermal performance of a counter flow heat exchanger. The potential for external heat transfer is the temperature difference between the ambient and the fluids of the heat exchanger. They developed equations for predicting the temperature and the effectiveness of both the fluids. This model is very similar to the models developed by Barron [8] and Chowhury and Sarangi [9], however this model is superior in comparison to those models in the sense that in this model both the fluids can be simultaneously subjected to external heating if needed. According to their model the effectiveness of the fluids depended on NTU, heat capacity ratio, the ambient temperature and the thermal resistance between the individual fluids and the ambient. Using this model they studied the effect of external heat transfer on the effectiveness of the fluids for several ambient temperatures and various values of thermal resistance between the individual fluids and the ambient. Ameel and Hevawitharna [11] observed temperature cross under certain situations of external heating. Through their analysis they concluded that there was no use in increase in NTU once temperature cross was observed in a heat exchanger at a particular level of external heat transfer. The maximum value of NTU that can be achieved in heat exchanger without observing temperature cross decreased with reduction in the thermal resistance between the individual fluids and the ambient for a particular ambient temperature. These authors defined NTU using the heat capacity of the cold fluid rather than using the fluid with minimum heat capacity. Moreover, the heat capacity ratio is taken as the ratio of the heat capacity of the cold fluid to that of the hot fluid. Thus, NTU and C_r used in this model do not correspond to the same NTU and C_r of the conventional ε-NTU relationships unless the cold fluid has the lowest heat capacity.

Ameel [12] modeled a parallel flow heat exchanger in which both the fluids interacted thermally with the ambient. In this model the fluids were subjected to a constant temperature. Analytical equations were developed by Ameel [12] for estimating the axial temperature of the fluids. The effectiveness as well as the axial temperature of the fluids was found to depend on NTU, heat capacity ratio (C_r), ambient temperature and the thermal resistances between the individual fluids and the ambient. Ameel [12] used this model to analyze a balanced flow heat exchanger. With increase in ambient temperature the effectiveness of the hot and cold fluid decreased and increased, respectively. For a particular heat exchanger operating at a specific ambient temperature, the temperature of the fluids approached that of the ambient with reduction in the thermal resistance between the individual fluids and the ambient. While defining operating parameters such as NTU and C_r, the heat capacity of the cold fluid was used instead of the lowest heat capacity among the fluids in the heat exchanger. This limited the applicability of this model to just balanced flow or whenever the cold fluid has the lowest heat capacity. On the other hand, the heat capacity of the hot fluid was used instead of the lowest heat capacity while calculating the maximum heat transfer possible in a heat exchanger (q_{max}) for use in estimating the hot fluid effectiveness.

Nellis and Pfotenhauer [13] developed analytical equations for calculating the hot and cold fluid effectiveness of a counter flow heat exchanger subjected to external heat flux. The effectiveness of the hot fluid reduced when either of the fluids was externally heated. The reduction in hot fluid effectiveness was greater when only the hot fluid was externally heated in contrast to the situation where only the cold fluid was externally heated. The equations for determining the axial temperature of the fluids were not provided in this paper. In their analysis NTU was defined with respect to the heat capacity of the hot fluid rather than with respect to the heat capacity of the fluid with the least heat capacity. Moreover, the heat capacity ratio was taken as the ratio of the heat capacity of the hot fluid to that of the cold fluid and not as the ratio of the minimum to maximum heat capacity. Thus, care needs to be taken while using these equations since the input parameters (NTU and C_r) are defined differently from that used in conventional ε-NTU equations. The external heat transferred to the individual fluids were made dimensionless using the term $UA(T_{h,i} - T_{c,i})$. This term, $UA(T_{h,i} - T_{c,i})$, does not have any physical significance with respect to the working of a heat exchanger. It may have been more appropriate to define external heat transfer with respect to either $UA\Delta T_{lm}$ or $C_{min}(T_{h,i} - T_{c,i})$.

A model for estimating the effect of uniform external heat flux on the thermal performance of a balanced parallel flow MCHX ($MCHX_{PF}$) was developed by Mathew and Hegab [14]. In this case the heat transfer between the fluids and the external heat source remained constant over the entire length of the $MCHX_{PF}$. They observed that the application of uniform external heat flux degraded and improved the effectiveness of the hot and cold fluids, respectively. These authors performed experiments on two MCHXs fabricated in silicon to verify the results of the model. Deionized water was used as the hot and cold fluid. The experiments were conducted for NTU ranging from 0.42 to 1.76 and the fluids were subjected to 0%, 5% and 10% external heating. For 0% external heating the effectiveness of the fluids ranged from 0.28 to 0.48. The hot fluid effectiveness for 5% and 10% external heating ranged from 0.23 to 0.43 and 0.18 to 0.38, respectively. On the cold fluid effectiveness varied from 0.33 to 0.53 and 0.38 to 0.58 for 5% and 10% external heating respectively. The results from the model were well within the uncertainty of the experimental results. To the knowledge of the authors this is the only paper that has experimentally analyzed the effect of external heat transfer on the effectiveness of a $MCHX_{PF}$.

Al-Dini and Zubair [15] recently developed analytical solutions for determining the effectiveness of the fluids of a parallel flow heat exchanger subjected to external heat flux. They analyzed two cases; in the first case the hot fluid was considered to have the lowest heat capacity and in the second case the cold fluid was taken as the fluid with the lowest heat capacity [15]. From their model the authors found the effectiveness of the hot fluid, irrespective of the heat capacity ratio, degraded whenever external heat flux was applied to each of the fluids individually. The degradation in the hot fluid effectiveness is greater when the hot fluid is externally heated. The opposite trend in the effectiveness of the cold fluid was observed when the fluids were subjected to external heat flux independently. For each of the above mentioned cases the authors only provided the equations for estimating the effectiveness of the fluid with the lowest heat capacity, i.e. the hot and cold fluid in the first and second case, respectively. Moreover, they did not develop equations for determining the axial temperature of the fluids which can be important in many applications involving heat exchangers. Al-Dini and Zubair [15] have also provided separate equations for determining the effectiveness of the fluids when NTU is zero. However, these equations do not appear to properly represent the physical situation in a heat exchanger when NTU is zero. These issues limit the applicability of this model.

Recently Mathew and Hegab [16] numerically modeled a balanced flow $MCHX_{PF}$ with internal heat generation and external heat transfer. Internal heat generation was considered in this study to account for the conversion of pumping power into heat, an effect that could be significant for microchannel flows. Heat transfer between the ambient and the fluids due to lack of proper thermal insulation is the external heat transfer considered in this particular study. The fluids are subjected to a constant temperature since the ambient is the external heat source. The individual effect of internal heat generation is to always degrade and improve the effectiveness of the hot and cold fluids, respectively. On the other hand, the presence of external heat transfer alone can either improve or degrade the effectiveness of the fluids depending on the ambient temperature. When the ambient temperature is above that of the inlet temperature of the hot fluid the effectiveness of the hot and cold fluid degraded and improved, respectively. The opposite effect in the effectiveness of the fluids was observed when the ambient temperature is below the inlet temperature of the cold fluid. The combination of these two effects can either improve or degrade the effectiveness of the fluids depending on the net heat transferred to the individual fluids due to these two phenomena.

Mathew and Hegab [17] also modeled a counter flow MCHX ($MCHX_{CF}$) subjected to uniform external heat flux. The model can be used for both balanced and unbalanced flow $MCHX_{CF}$. According to the model the effect of external heating would degrade the effectiveness of the hot fluid while improving the effectiveness of the cold fluid. The opposite trend in the effectiveness of the fluids was noticed when the fluids were subjected to external cooling. The authors observed that while operating under unbalanced flow conditions the effectiveness depended not only on NTU and heat capacity ratio (C_r) but also on the fluid with the minimum heat capacity. Under unbalanced flow conditions, at a particular NTU and heat capacity ratio, the effectiveness of the fluids was found to be better in a $MCHX_{CF}$ in which the hot fluid has the lowest heat capacity than in a $MCHX_{CF}$ in which the cold fluid has the lowest heat capacity.

The ε-NTU relationships for a $MCHX_{PF}$ subjected to uniform external heat flux are developed in this paper. This particular model has been developed in such a way that it can be used for a $MCHX_{PF}$ with either balanced or unbalanced flow. The temperature of fluids at any axial location can also be calculated using this model. In situations where a $MCHX_{PF}$ is used for handling cells or conducting chemical reactions the knowledge of local temperature might be as useful as the effectiveness of the individual fluids. In addition, this thermal model has the advantage that it can be used when the individual fluids are subjected to either equal or unequal amounts of external heat transfer. The operating parameters, such as NTU, and external heat transfer, used in this model have been defined in a general way thereby making it easy to use the equations associated with this model.

The effectiveness of a $MCHX_{CF}$ is always higher than heat exchangers with other types of flow arrangements. However, it suffers from axial heat conduction due to which its effectiveness can become comparable to that of a $MCHX_{PF}$. Therefore, the authors of this paper feel that at the microscale a $MCHX_{PF}$ could be as important as a $MCHX_{CF}$. Several review articles about MCHXs have been published in the recent past; these articles provide insights about their current design trends as well as applications [18,19].

2. Theory

The theoretical model of the $MCHX_{PF}$ subjected to uniform external heat flux has been developed in this section of the paper. A differential element of such a $MCHX_{PF}$ is presented in Fig. 1. While developing this model certain assumptions were made and they are mentioned below.

1) $MCHX_{PF}$ operates under steady state conditions.
2) The temperature of the fluids varies only in the axial direction.
3) The flow is considered to be thermally and hydrodynamically fully developed between the inlet and outlet.
4) Slip boundary condition and rarefaction effects are assumed to be nonexistent in the microchannels (Kn < 0.001).
5) The fluids do not undergo phase change while flowing through the channels.
6) Influence of effects such as longitudinal heat conduction through the fluids and wall, viscous dissipation and flow maldistribution are considered to be negligible.
7) The thermophysical properties of the fluids are assumed to be constant over the length of the $MCHX_{PF}$.

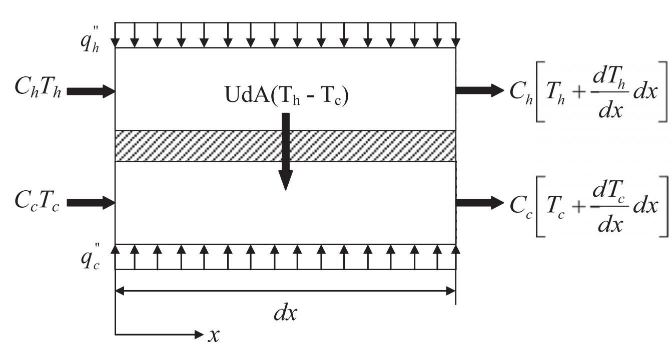

Fig. 1. Schematic representation of the energy balance of a differential element of the $MCHX_{PF}$ considered in this study.

8) The ends of the wall separating the fluids are considered to be insulated.
9) The flow is assumed to be incompressible ($Ma < 0.3$).

Based on these assumptions the governing equations, in nondimensional form, have been developed and provided in equations (1) and (2). The first term on the left hand side of equations (1) and (2) represents the axial variation of fluid temperature. The heat transferred between the fluids is represented by second term on the left hand side of these equations. The term on the right hand side of equations (1) and (2) accounts for external heat transfer. The boundary conditions for these equations are the inlet temperature of the fluids and they have been provided in nondimensional form in equations (3) and (4).

$$\frac{d\theta_h}{dZ} + C_{Rh}\mathrm{NTU}(\theta_h - \theta_c) = C_{Rh}Q_h \tag{1}$$

$$\frac{d\theta_c}{dZ} - C_{Rc}\mathrm{NTU}(\theta_h - \theta_c) = C_{Rc}Q_c \tag{2}$$

$$\theta_h|_{Z=0} = 1 \tag{3}$$

$$\theta_c|_{Z=0} = 0 \tag{4}$$

C_{Rh} and C_{Rc} represent the individual heat capacity ratios. In a balanced flow $MCHX_{PF}$ both C_{Rh} and C_{Rc} are same and equal to unity. In an unbalanced flow $MCHX_{PF}$ if the hot fluid has the lowest heat capacity then C_{Rh} would be equal to unity and C_{Rc} would be same as the conventional heat capacity ratio ($C_{Rc} = C_h/C_c = C_r$). On the other hand, if the cold fluid has the lowest heat capacity among the fluids in a $MCHX_{PF}$ then C_{Rh} would be same as the conventional heat capacity ratio ($C_{Rh} = C_c/C_h = C_r$) and C_{Rc} would be equal to unity. The parameter Q in equations (1) and (2) is a nondimensional external heat transfer parameter. Positive values of Q represent external heating of the fluids while negative values of Q would represent external cooling of the fluids. Equations (1) and (2) constitute a system of equations which need to be solved simultaneously. The mathematical technique for solving this system of ODEs has been provided by Wylie [20]. Equations (5) and (6) are the solutions of the equations (1) and (2). The constants in equations (5) and (6) were determined using the boundary conditions presented in equations (3) and (4).

$$\theta_h = K_1 + K_2\, e^{-\mathrm{NTU}(C_{Rh}+C_{Rc})Z} + \left(\frac{C_{Rh}C_{Rc}}{C_{Rh}+C_{Rc}}\right)(Q_h + Q_c)Z \tag{5}$$

$$\theta_c = K_1 - \left(\frac{C_{Rc}}{C_{Rh}}\right)K_2\, e^{-\mathrm{NTU}(C_{Rh}+C_{Rc})Z} + \left(\frac{C_{Rh}C_{Rc}}{C_{Rh}+C_{Rc}}\right)(Q_h+Q_c)Z + \left(\frac{Q_cC_{Rc}-Q_hC_{Rh}}{\mathrm{NTU}(C_{Rh}+C_{Rc})}\right) \tag{6}$$

where, $K_1 = \left(\frac{C_{Rh}}{C_{Rh}+C_{Rc}}\right)\left(\frac{C_{Rc}}{C_{Rh}} - \frac{Q_cC_{Rc}-Q_hC_{Rh}}{\mathrm{NTU}(C_{Rh}+C_{Rc})}\right)$

$K_2 = \left(\frac{C_{Rh}}{C_{Rh}+C_{Rc}}\right)\left(1 + \frac{Q_cC_{Rc}-Q_hC_{Rh}}{\mathrm{NTU}(C_{Rh}+C_{Rc})}\right)$

Careful examination of equations (1) and (2) will reveal that the governing equations are independent of the profile of the channels. The influence of the channel profile on the effectiveness of a $MCHX_{PF}$ is embedded in the nondimensional parameter NTU. Therefore the solutions of equations (1) and (2), i.e. equations (3) and (4), can be used irrespective of the profile of the channels of the $MCHX_{PF}$. The effectiveness of the fluids has been defined as the ratio of the net heat gained/lost (q) by the individual fluid to the maximum heat transfer (q_{max}) possible in a $MCHX_{CF}$ free of external heat transfer. The effectiveness of the fluids is determined using the inlet and outlet temperatures of the fluids as well as the heat capacity ratios (C_{Rh} and C_{Rc}) as shown in equations (7) and (8).

$$\varepsilon_h = \frac{q_h}{q_{max}} = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{1 - \theta_h|_{Z=1}}{C_{Rh}} \tag{7}$$

$$\varepsilon_c = \frac{q_c}{q_{max}} = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{\theta_c|_{Z=1}}{C_{Rc}} \tag{8}$$

Equations (7) and (8) cannot be used when NTU is zero or infinite. A $MCHX_{PF}$ may never be operated at these NTUs, however in order to make this theoretical treatment complete the effectiveness of the fluids at these NTUs is provided in Table 1. These values are obtained by applying L'Hospital's rule to equations (7) and (8).

Equations (5)–(8) can be used even when the $MCHX_{PF}$ is free of external heat transfer by substituting zero for Q_h and Q_c. Upon these substitutions equations (5)–(8) can be written as follows:

$$\theta_h = \left(\frac{C_{Rc}}{C_{Rh}+C_{Rc}}\right)\left(1 + \frac{C_{Rh}}{C_{Rc}}\, e^{-\mathrm{NTU}(C_{Rh}+C_{Rc})Z}\right) \tag{9}$$

$$\theta_c = \left(\frac{C_{Rc}}{C_{Rh}+C_{Rc}}\right)\left(1 - e^{-\mathrm{NTU}(C_{Rh}+C_{Rc})Z}\right) \tag{10}$$

$$\varepsilon_h = \varepsilon_c = \left(\frac{1}{C_{Rh}+C_{Rc}}\right)\left(1 - e^{-\mathrm{NTU}(C_{Rh}+C_{Rc})}\right) \tag{11}$$

These equations are same as that provided by Shah and Sekulic [6] for a parallel flow heat exchanger free of external heat transfer.

The heat transferred between the fluids is determined using the local temperatures of the fluids. The general equation for determining the total heat transferred between the fluids is presented in equation (12) and equation (13) represents its final form.

$$Q^*_{h,c} = \int_0^1 dQ^*_{h,c} = \int_0^1 \mathrm{NTU}(\theta_h - \theta_c)dZ \tag{12}$$

$$Q^*_{h,c} = \left(\frac{K_2}{C_{Rh}}\right)\left(1 - e^{-\mathrm{NTU}(C_{Rh}+C_{Rh})}\right) - \left(\frac{Q_cC_{Rc} - Q_hC_{Rh}}{C_{Rh}+C_{Rc}}\right) \tag{13}$$

Fig. 2 represents the several heat transfer paths to and from the whole of the $MCHX_{PF}$. In comparison, only the heat transfer paths of a differential element of the $MCHX_{PF}$ were presented in Fig. 1. The net heat gained/lost by the individual fluids (Q_{net}) between the inlet and outlet sections of the $MCHX_{PF}$ can be determined by applying the concept of energy balance to each of the fluids. The control volume over which the energy balance is performed is represented by the dotted lines in Fig. 2. The thermal energy advected through the inlet section ($C_h\theta_{h,i}$) and the external heat transfer (Q_h) are the energy inputs to the hot fluid. For the cold fluid the energy inputs comprise of the thermal energy advected through its inlet section ($C_c\theta_{c,i}$), the external heat transfer (Q_c) and the heat transfer between the fluids ($Q^*_{h,c}$). The heat transfer between the fluids ($Q^*_{h,c}$) and the thermal energy advected through the exit section of

Table 1
Effectiveness of the fluids when NTU = 0 and ∞.

NTU	Hot fluid	Cold fluid
0	$-Q_h$	Q_c
∞	$\frac{1}{C_{Rh}+C_{Rc}} - \frac{C_{Rc}(Q_h+Q_c)}{C_{Rh}+C_{Rc}}$	$\frac{1}{C_{Rh}+C_{Rc}} + \frac{C_{Rh}(Q_h+Q_c)}{C_{Rh}+C_{Rc}}$

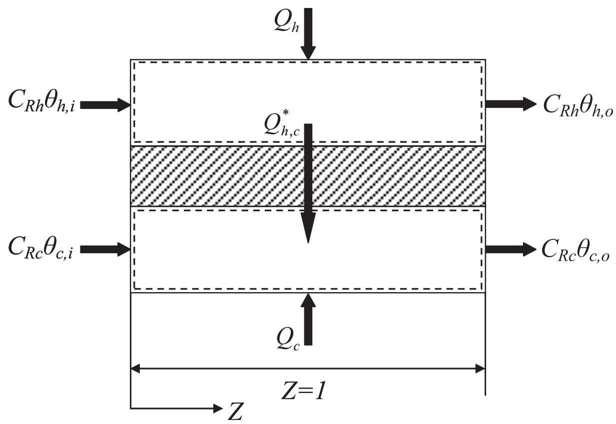

Fig. 2. Schematic representation of the energy balance of the individual fluids of the $MCHX_{PF}$ considered in this study.

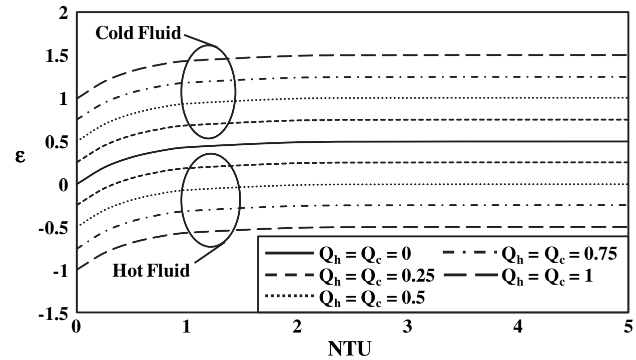

Fig. 3. ε-NTU relationship of a balanced flow $MCHX_{PF}$ ($C_{Rh} = C_{Rc} = 1$).

the hot fluid ($C_h\theta_{h,o}$) make up the energy transfers out of the hot fluid. Similarly the thermal energy advected through the cold fluid outlet section ($C_h\theta_{h,i}$) is the only energy transfer from the cold fluid. Mathematically, the net heat gained/lost (Q_{net}) can be represented as provided in equations (14) and (15).

$$Q_{h,net} = \varepsilon_h = Q^*_{h,c} - Q_h \tag{14}$$

$$Q_{c,net} = \varepsilon_c = Q^*_{h,c} + Q_c \tag{15}$$

It was mentioned earlier that several phenomena that could be significant in a MCHX are not included in the conventional ε-NTU relationship and viscous heating is one such phenomena [6]. Viscous heating becomes significant in the presence of high pumping power [15,21]. Pumping power is a function of pressure drop and volumetric flow rate. Pressure drop increases with decrease in the size of the channel for a specific flow rate and thus it can become significant in MCHXs. The effect of viscous heating is to always raise the temperature of the fluids. The heat generated due to viscous heating is constant over the length of the channels [15,21]. Therefore the presence of viscous heating is exactly similar to that of subjecting the fluids to uniform heat flux. Thus the model developed in this paper has the advantage that it can be used for modeling a $MCHX_{PF}$ with viscous heating as well.

The model developed in this section is based on the assumption that $Kn < 10^{-3}$. Thus, this model can be applied to all heat exchangers in which $Kn < 10^{-3}$. For almost all macroscale heat exchangers this assumption is valid and thus the equations developed in this paper can be used even for a macroscale heat exchanger that is subjected to external heating.

3. Results and discussion

In this section the effect of external heat transfer on the thermal performance of a $MCHX_{PF}$ has been studied using the equations developed in this paper. It has been mentioned that the equations formulated in this paper can be used for predicting the temperature as well as the effectiveness of the fluids that are being heated or cooled by an external heat source. However, in this section due to space limitations only the influence of external heating has been analyzed.

Fig. 3 represents the ε-NTU relationship of a balanced flow $MCHX_{PF}$ subjected to external heating. The effectiveness of fluids in a balanced flow $MCHX_{PF}$ without external heating is represented by the solid line. In this figure NTU is varied between zero and five. This range of NTU is selected because the effectiveness of a $MCHX_{PF}$, irrespective of the heat capacity ratio, would reach an asymptotic value when NTU is close to five. Thus, for most practical applications a $MCHX_{PF}$ would be designed for NTU values lower than five. For specific nondimensional external heat transfer parameters (Q_h and Q_c) the effectiveness of the cold fluid is greater than that of the hot fluid over the entire range of NTU. This can be attributed to the increase in the temperature of the fluids that occurs due to external heating.

According to the plots in Fig. 3 the effectiveness of both fluids initially increased with increase in NTU for a specific Q_h and Q_c. Similar trend can also be observed in a $MCHX_{PF}$ free of external heating. In order to understand this trend the effect of increase in NTU on Q_h, Q_c and the nondimensional heat transferred between the fluids ($Q^*_{h,c}$) have to be analyzed. Q_h and Q_c are functions of external heat transfer ($q_{ext} = q''A$) and the maximum heat transfer possible in a heat exchanger (q_{max}) while $Q^*_{h,c}$ depends on the heat transferred between the fluids ($q_{h,c}$) and the maximum heat transfer possible in a heat exchanger (q_{max}). In a $MCHX_{PF}$, NTU can be raised by increasing the heat transfer surface area or by decreasing the flow rate. Increases in heat transfer surface area do not change the maximum heat transfer possible in a heat exchanger (q_{max}). However, an increase in heat transfer surface area is associated with a corresponding decrease in external heat flux in order to keep external heat transfer (q_{ext}) constant and thereby prevent any changes in Q_h and Q_c. Moreover, the increase in heat transfer surface area improved the heat transfer between the fluids ($q_{h,c}$). Thus, when NTU is increased by increasing the heat transfer surface area, the external heat transfer (q_{ext}) and maximum heat transfer possible in a MCHX (q_{max}) remains the same but the heat exchanged between the fluids ($q_{h,c}$) improves and this results in the increase in the effectiveness of the fluids. The above mentioned effect of NTU on the nondimensional heat transfer between the fluids ($Q^*_{h,c}$), ε_h and ε_c is quantitatively presented in Table 2 for $Q_h = Q_c = 0$, 0.25 and 0.5. On the other hand, if the NTU is raised by reducing the flow rate, the maximum heat transfer possible between the fluids (q_{max}) must decrease to keep Q_h and Q_c constant. This leads to the reduction in external heat transfer to the fluids (q_{ext}) in order to keep Q_h and Q_c constant. Therefore, for a given geometry the external heat flux must decrease in order to maintain Q_h and Q_c constant. In addition, a reduction in flow rate brings about an increase in the nondimensional heat transfer between the fluids ($Q^*_{h,c}$) due to increased residence time. Therefore these two effects combined result in an increase in effectiveness with decrease in NTU. The data provided in Table 2 applies even when NTU is altered by changing the flow rate.

Table 2
Effect of NTU on $Q^*_{h,c}$, ε_h and ε_c for a balanced flow $MCHX_{PF}$ ($C_{Rh} = C_{Rc} = 1$, $Q_h = Q_c$).

NTU	$Q^*_{h,c}$	Q_h	Q_c	$Q_{net,h} = \varepsilon_h$	$Q_{net,c} = \varepsilon_c$
0	0.000	0	0	0	0
0.25	0.197	0	0	0.197	0.197
0.5	0.316	0	0	0.316	0.316
0.75	0.388	0	0	0.388	0.388
1	0.432	0	0	0.432	0.432
2	0.491	0	0	0.491	0.491
3	0.500	0	0	0.500	0.500
4	0.500	0	0	0.500	0.500
0	0.000	0.25	0.25	−0.250	0.250
0.25	0.197	0.25	0.25	−0.053	0.447
0.5	0.316	0.25	0.25	0.066	0.566
0.75	0.388	0.25	0.25	0.138	0.638
1	0.432	0.25	0.25	0.182	0.682
2	0.491	0.25	0.25	0.241	0.741
3	0.500	0.25	0.25	0.250	0.750
4	0.500	0.25	0.25	0.250	0.750
0	0.000	0.5	0.5	−0.500	0.500
0.25	0.197	0.5	0.5	−0.303	0.697
0.5	0.316	0.5	0.5	−0.184	0.816
0.75	0.388	0.5	0.5	−0.112	0.888
1	0.432	0.5	0.5	−0.066	0.932
2	0.491	0.5	0.5	−0.01	0.991
3	0.500	0.5	0.5	0.00	1.00
4	0.500	0.5	0.5	0.00	1.00

From Table 2 it can also be noticed that for a particular NTU, the nondimensional heat transfer between the fluids ($Q^*_{h,c}$) of a $MCHX_{PF}$ subjected to external heat transfer is same as that in a $MCHX_{PF}$ free of external heating. This is because the addition of equal amounts of heat from the external heat source, i.e. $Q_h = Q_c$, to the fluids that are flowing in the same direction will raise their axial temperatures by the same amount. Thus, at a particular NTU, the difference in axial temperatures of the fluids as well as the nondimensional heat transferred between the fluids ($Q^*_{h,c}$) remain the same as that in a $MCHX_{PF}$ free of external heating. On the other hand, whenever Q_h and Q_c of a $MCHX_{PF}$ are unequal, the nondimensional heat transfer between the fluids ($Q^*_{h,c}$) of such a heat exchanger will not be same as that in a $MCHX_{PF}$ without external heat transfer.

From Fig. 3 it can be noticed that when NTU is zero the hot fluid effectiveness is equal to negative of the hot fluid nondimensional external heat transfer parameter ($-Q_h$). This is because the nondimensional heat transferred between the fluids ($Q^*_{h,c}$) is zero when NTU is zero, therefore the heat (Q_h) supplied to the hot fluid by the external heat source remains in it. A similar trend is observed in the effectiveness of the cold fluid when NTU is zero. The reason for this is same as that explained for the hot fluid.

Fig. 4 contains the temperature profile of the fluids for NTU = 5 and $Q_h = Q_c = 0$, 0.25, 0.5, 0.75 and 1. From this figure it can be noticed that the location at which the temperature of the fluids start to become equal is the same for all cases of external heating. This location has been marked using a vertical line in Fig. 4. However, the temperature at this location is dependent on the amount of external heating. It was mentioned earlier that if the fluids of a $MCHX_{PF}$ are subjected to equal amounts of external heating then the difference in the axial temperature of the fluids at any location would be independent of the amount of external heating and remain same as that in $MCHX_{PF}$ without external heating. Therefore the location at which the temperature of the fluids becomes equal is also independent of the amount of external heating. In a $MCHX_{PF}$ the difference in the temperature of the fluids is zero starting from the location where they become equal until the exit section.

Figs. 5 and 6 represent the effect of external heat transfer on the thermal performance of an unbalanced flow $MCHX_{PF}$. An

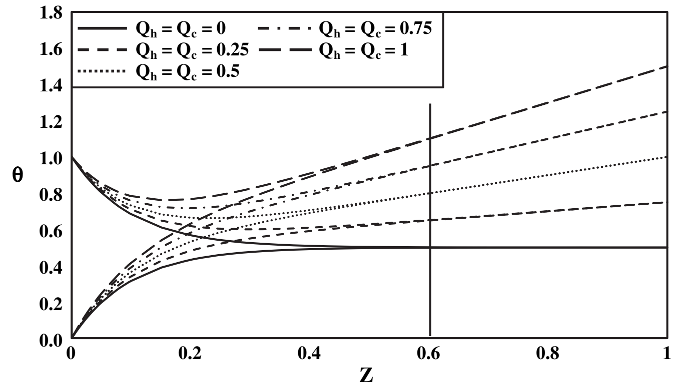

Fig. 4. Temperature profile of the hot and cold fluid in a balanced flow $MCHX_{PF}$ at NTU = 5 ($C_{Rh} = C_{Rc} = 1$).

unbalanced flow $MCHX_{PF}$ can be operated with either of the fluids as the one with the lowest heat capacity. The thermal performance of a $MCHX_{PF}$ free of external heat transfer is dependent only on the conventional heat capacity ratio (C_r) and NTU; it is not dependent on the fluid with the lowest heat capacity. However the same cannot be said about an unbalanced flow $MCHX_{PF}$ subjected to external heat transfer and this is shown in Figs. 5 and 6. In these figures the conventional heat capacity ratio (C_r) is maintained at 0.5. The hot fluid heat capacity in Fig. 5 is lower than that of the cold fluid. Therefore for Fig. 5 $C_{Rh} = 1$ and $C_{Rc} = 0.5$. In Fig. 6 the heat capacity of the hot fluid is greater than that of the cold fluid and mathematically this can be represented as $C_{Rh} = 0.5$ and $C_{Rc} = 1$. For both these figures the nondimensional external heat transfer parameters are kept equal, i.e. $Q_h = Q_c = 0$, 0.25, 0.5, 0.75, 1.

In Figs. 5 and 6 the effectiveness of the hot and cold fluid in a $MCHX_{PF}$ without external heating is shown by the solid line. The effectiveness of the hot fluid in these figures decreased with increase in nondimensional external heat transfer parameters for a specific NTU. On the other hand the cold fluid effectiveness improved with increase in nondimensional external heat transfer parameters for a particular NTU. For a specific Q_h and Q_c the effectiveness of the fluids initially increased with increase in NTU. When either of the $MCHX_{PF}$ analyzed in Figs. 5 and 6 is operated at zero NTU the effectiveness of the hot and cold fluids is equal to $-Q_h$ and Q_c, respectively. The reasons for these are same as that explained earlier for the balanced flow $MCHX_{PF}$.

On comparing Figs. 5 and 6 it can be noticed that for a specific NTU, Q_h and Q_c, the effectiveness of the fluids when the hot fluid has the lowest heat capacity is higher than the effectiveness of the fluids when the cold fluid has the lowest heat capacity. From

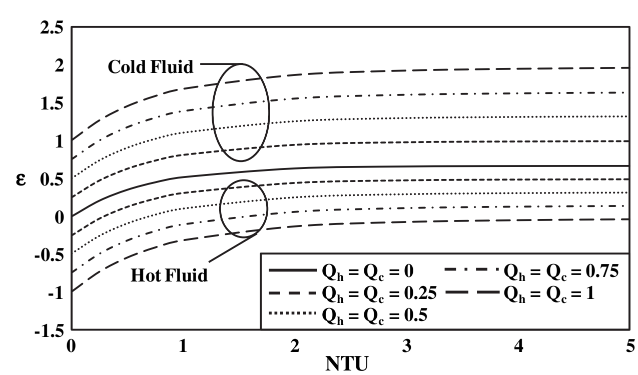

Fig. 5. ε-NTU relationship of an unbalanced flow $MCHX_{PF}$ ($C_{Rh} = 1$, $C_{Rc} = 0.5$).

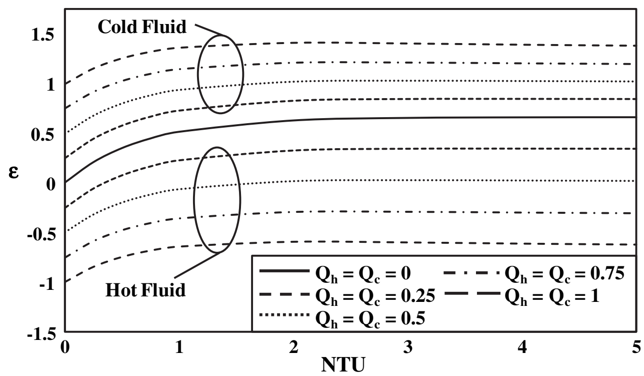

Fig. 6. ε-NTU relationship of an unbalanced flow $MCHX_{PF}$ ($C_{Rh} = 0.5$, $C_{Rc} = 1$).

equations (14) and (15) it can be seen that the effectiveness of the fluids depends on the heat transferred between the fluids ($Q^*_{h,c}$) and the nondimensional external heat transfer parameters (Q_h and Q_c). Therefore for all the cases where Q_h and Q_c are kept constant the effectiveness depends only on the heat transferred between the fluids ($Q^*_{h,c}$). Among the two unbalanced flow conditions ($C_{Rh} = 1$ and $C_{Rc} = C_r$ and $C_{Rh} = C_r$ & $C_{Rc} = 1$) the heat transferred between the fluids ($Q^*_{h,c}$), for a specific NTU, Q_h, and Q_c, is greater when the hot fluid has the lowest heat capacity. Fig. 7 depicts the variation in heat transferred between the fluids with respect to NTU for both cases of unbalanced flows with $Q_h = Q_c = 0.25$. Thus the effectiveness of the fluids is higher when the hot fluid has the lowest heat capacity.

In a $MCHX_{PF}$ free of external heating temperature cross over cannot happen [6]. However, the presence of external heating can cause temperature cross over in a $MCHX_{PF}$. Temperature cross over does not occur in all the parallel flow MCHXs that are subjected to external heat transfer. One situation where temperature cross over can occur is when a balanced flow $MCHX_{PF}$ in which Q_c is greater than Q_h. In Fig. 8, temperature profiles of the fluids at several NTUs (2, 5, and 10) are shown for a balanced flow $MCHX_{PF}$ in which $Q_h = 0$ and $Q_c = 1$. Temperature cross over is clearly evident in this figure for all the NTUs considered. The location of temperature cross over (Z_{cr}) is determined graphically from the temperature profiles as shown in Fig. 8. The location of temperature cross over for the above mentioned NTUs is provided in Table 3. From Fig. 4 it can be seen that temperature of the cold fluid never became greater than that of the hot fluid. In comparison, in Fig. 8 it can be noticed that the temperature of the cold fluid became greater than that of the hot fluid after the location of temperature cross over ($Z = Z_{cr}$).

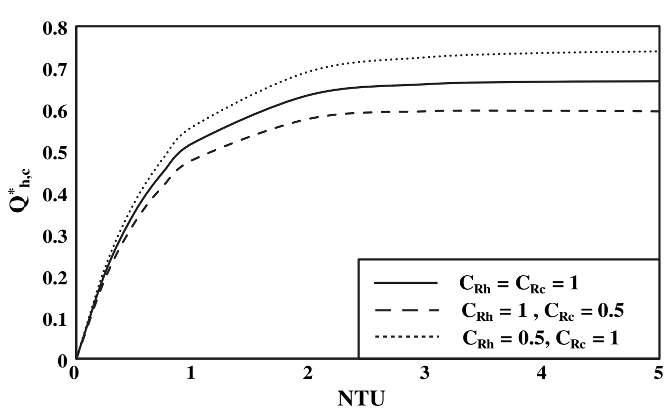

Fig. 7. Variation of $Q^*_{h,c}$ with NTU of a $MCHX_{PF}$ ($Q_h = Q_c = 0.25$).

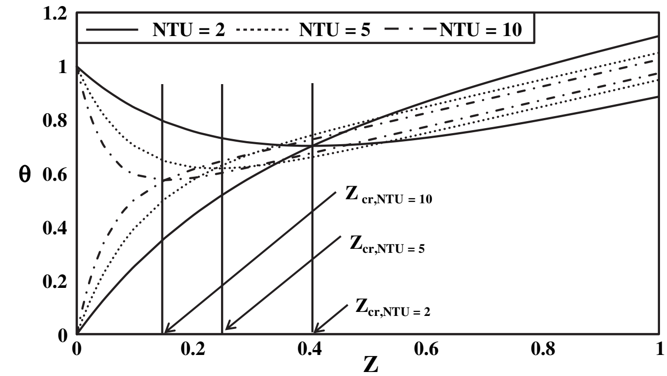

Fig. 8. Temperature profile of the hot and cold fluid in a balanced flow $MCHX_{PF}$ at NTU = 2, 5 and 10 ($C_{Rh} = C_{Rc} = 1$, $Q_h = 0$, $Q_c = 1$).

Temperature cross over occurred in this $MCHX_{PF}$ because the individual fluids have been unequally heated by the external heat source. Therefore, when both fluids started moving towards the outlet ($Z = 1$) from the location of temperature cross over ($Z = Z_{cr}$) the amount of external heating supplied to the cold fluid per unit length was greater than that supplied to the hot fluid and this caused the temperature of the cold fluid to rise above that of the hot fluid. In addition, on comparing the temperature profiles of the fluids at NTU = 2, 5, and 10 it can be noticed that the difference in temperature of the fluids at any location between the point of temperature cross over ($Z = Z_{cr}$) and the outlet section ($Z = 1$) decreased with increase in NTU. With increase in NTU the thermal resistance between the fluids reduces that result in smaller temperature difference as observed. Another inference that can be drawn from this second observation is that with increases in NTU the difference between the temperatures of the fluids at any location between the location of temperature cross over ($Z = Z_{cr}$) and the outlet section ($Z = 1$) will decrease and ultimately eliminate temperature cross over. Temperature cross over can also occur in an unbalanced flow $MCHX_{PF}$ ($C_{Rh} < 1$, and $C_{Rc} = 1$) subjected to external heating. The amount of external heat transfer to the fluids of this $MCHX_{PF}$ can either be equal or the external heat supplied to the cold fluid should be greater than that supplied to the hot fluid. Fig. 9 represents the temperature profile of fluids in an unbalanced flow $MCHX_{PF}$, $C_{Rh} = 0.25$ and $C_{Rc} = 1$, at several NTUs (2, 5 and 10) subjected to equal amounts of external heat transfer ($Q_h = Q_c = 0.5$). The location of temperature cross over was graphically determined from Fig. 9 for these values of NTU and they have been presented in Table 4. When the cold fluid has the lowest heat capacity its rise in temperature beyond the location of temperature cross over ($Z = Z_{cr}$) would be greater than that experienced by the hot fluid and thus the occurrence of temperature cross over. The effect of rise in NTU on Z_{cr}, $Q^*_{h,c,cr-1}$, ε_h and ε_c is shown in Table 3. The difference in the temperature of the fluids between the location of temperature cross over ($Z = Z_{cr}$) and the outlet section ($Z = 1$) decreased with increase in NTU. This behavior is similar to that observed in Fig. 8.

Table 3
Effect of NTU on Z_{cr}, $Q^*_{h,c,0-cr}$, $Q^*_{h,c}$, ε_h and ε_c of a balanced flow $MCHX_{PF}$ ($C_{Rh} = C_{Rc} = 1$, $Q_h = 0$, $Q_c = 1$).

NTU	Z_{cr}	$Q^*_{h,c,0-cr}$	$Q^*_{h,c}$	ε_h	ε_c
1	0.55	0.225	0.149	0.149	1.149
5	0.24	0.476	0.045	0.045	1.045
20	0.093	0.497	0.013	0.013	1.013

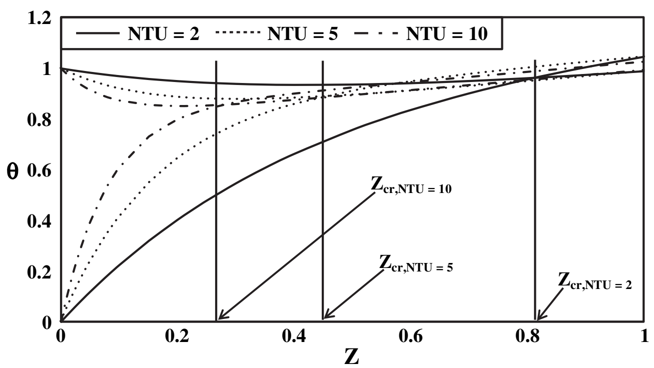

Fig. 9. Temperature profile of the hot and cold fluid in an unbalanced flow $MCHX_{PF}$ at NTU = 2, 5 and 10 (C_{Rh} = 0.25, C_{Rc} = 1, $Q_h = Q_c$ = 0.5).

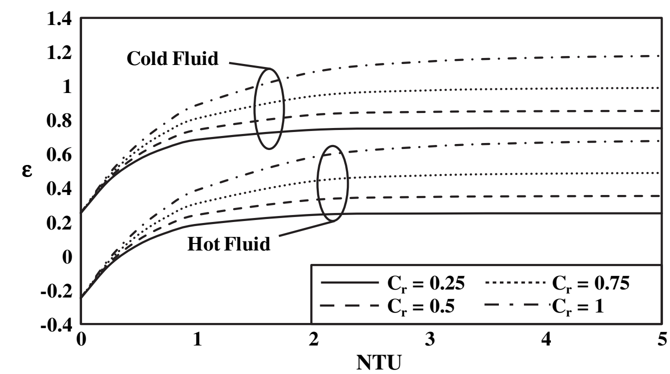

Fig. 10. Effect of C_r on the ε-NTU relationship of an unbalanced flow $MCHX_{PF}$ (C_{Rh} = 1, $C_{Rc} = C_r$, $Q_h = Q_c$ = 0.25).

From the plots of Figs. 8 and 9 it can be seen that between the inlet section ($Z = 0$) of the fluids and the location of temperature cross over ($Z = Z_{cr}$) the cold fluid was heated by the hot fluid. Beyond this point the hot fluid was heated by the cold fluid. The effectiveness of the fluids for the NTU values of 2, 5 and 10 is shown in Tables 3 and 4. The location of temperature cross over (Z_{cr}), the heat transfer between $Z = 0$ and $Z = Z_{cr}$ ($Q^*_{h,c,0-cr}$) as well as the hot and cold fluid effectiveness are provided in Tables 3 and 4. It can be seen from Tables 3 and 4 that the effectiveness of the fluids decreased with increase in NTU in the presence of temperature cross over. This result is because the net heat transferred between the fluids decreased with increase in NTU. The heat transferred between the fluids from $Z = 0$ and $Z = 1$ ($Q^*_{h,c}$) is the sum of the heat transferred between the fluids from $Z = 0$ to $Z = Z_{cr}$ ($Q^*_{h,c,0-cr}$) and the heat transferred between the fluids from $Z = Z_{cr}$ to $Z = 1$ ($Q^*_{h,c,cr-1}$). Increase in the heat transferred between the fluids from $Z = 0$ to $Z = Z_{cr}$ ($Q^*_{h,c,0-cr}$) with increase in NTU was in part due to the reduction in q''_c. The heat transferred between the fluids from $Z = 0$ to $Z = Z_{cr}$ ($Q^*_{h,c,0-cr}$) also improved either due to increased heat transfer surface area or due to increased residence time depending on technique used to increase the NTU. Even the heat transferred between the fluids from $Z = Z_{cr}$ to $Z = 1$ ($Q^*_{h,c,cr-1}$) increased with increase in NTU even though the difference in the local temperatures of the fluids between $Z = Z_{cr}$ and $Z = 1$ reduced with increase in NTU. This result, is because with increase in NTU, the heat transferred between the fluids from $Z = Z_{cr}$ to $Z = 1$ ($Q^*_{h,c,cr-1}$) improved either due to increased surface area or due to increased residence time. The improvement in the heat transferred between the fluids from $Z = Z_{cr}$ to $Z = 1$ ($Q^*_{h,c,cr-1}$) was greater than the improvement in the heat transferred from $Z = 0$ to $Z = Z_{cr}$ ($Q^*_{h,c,0-cr}$) and this reduced the net heat transferred between the fluids from $Z = 0$ to $Z = 1$ ($Q^*_{h,c}$) with increase in NTU and thereby reducing the effectiveness of the fluids with rise in NTU.

Fig. 10 represents the effect conventional heat capacity ratio (C_r) on the ε-NTU relationship of a $MCHX_{PF}$ subjected to uniform external heat flux. In Fig. 10 the hot fluid has the lowest heat capacity, i.e. $C_{Rh} = 1$. The cold fluid has the highest heat capacity and thus $C_{Rc} = C_r$. From this figure it can be noticed that the effectiveness of the fluids increased with reduction in the conventional heat capacity ratio (C_r) for a particular value of NTU. This is mainly due to the improvement in the heat transferred between the fluids ($Q^*_{h,c}$). The trend observed in Fig. 10 is similar to that occurring in a $MCHX_{PF}$ free of external heating. According to the definition of the conventional heat capacity ratio (C_r) it can be reduced by either reducing the flow rate of the hot fluid ($C_h = C_{min}$) or by increasing the flow rate of the cold fluid ($C_c = C_{max}$). Therefore the mechanism by which $Q^*_{h,c}$ improves depends on whether C_h or C_c is altered to reduce C_r. Q_h and Q_c are kept constant for all values of C_r that are used in plotting Fig. 10. Q_h and Q_c depend on the maximum heat transfer possible in a heat exchanger (q_{max}) and the external heat transfer (q_{ext}) to the individual fluids. The maximum heat transfer possible in a heat exchanger (q_{max}) is not affected when the flow rate of the cold fluid ($C_c = C_{max}$) is changed in order to alter C_r. Thus, the external heat transfer (q_{ext}) to the individual fluids also remains unchanged since Q_h, Q_c and maximum heat transfer possible in a heat exchanger (q_{max}) are unchanged with changes in C_r. Therefore, the temperature rise of the cold fluid decreases when its flow rate is raised in order to decrease C_r since Q_h and Q_c are unaltered. This result brought about an increase in the heat transferred between the fluids and thereby improved the effectiveness of both fluids. Alternatively, any reduction in the heat capacity of the hot fluid to reduce C_r is accompanied by a reduction in q_{max}. This in turn causes q_{ext} to reduce since Q_h and Q_c are the kept same for all the plots in Fig. 10. Therefore, when C_h is reduced the heat supplied to the fluids from the external heat source reduces and it improved the heat transfer between the fluids. The influence of C_r on $Q^*_{h,c}$ is shown in Fig. 11 for NTU = 2, 5 and 10 for $Q_h = Q_c = 0.25$.

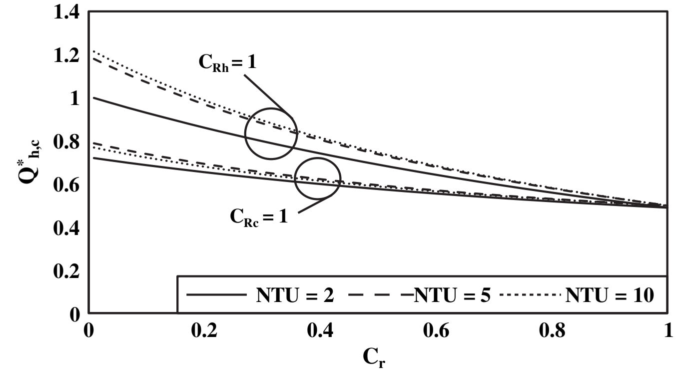

Fig. 11. Variation of $Q^*_{h,c}$ with C_r of a $MCHX_{PF}$ ($Q_h = Q_c = 0.25$).

Table 4
Effect of NTU on Z_{cr}, $Q^*_{h,c,0-cr}$, $Q^*_{h,c}$, ε_h and ε_c of an unbalanced flow $MCHX_{PF}$ ($C_{Rh} = 0.5$, $C_{Rc} = 1$, $Q_h = Q_c = 0.5$).

NTU	Z_{cr}	$Q^*_{h,c,0-cr}$	$Q^*_{h,c}$	ε_h	ε_c
2	0.815	0.678	0.544	0.045	1.045
5	0.46	0.772	0.546	0.046	1.464
10	0.274	0.792	0.524	0.024	1.024

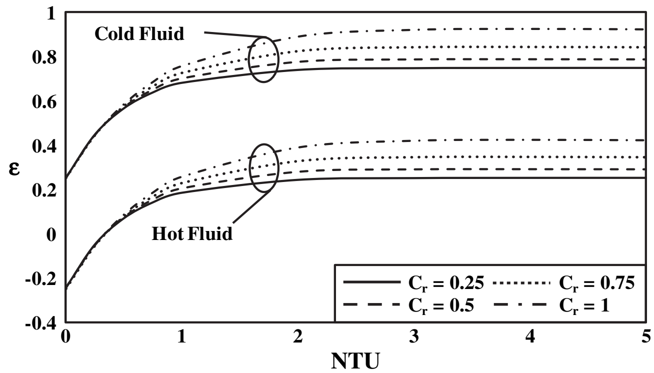

Fig. 12. Effect of C_r on the ε-NTU relationship of an unbalanced flow $MCHX_{PF}$ ($C_{Rh} = C_r$, $C_{Rc} = 1$, $Q_h = Q_c = 0.25$).

The effect of C_r on the ε-NTU relationship of a $MCHX_{PF}$ in which the cold fluid has the lowest heat capacity, i.e. $C_{Rc} = 1$, is show in Fig. 12. For this case the conventional heat capacity ratio (C_r) is same as C_{Rh}. For a particular NTU the effectiveness of the fluids increased with reduction in C_r. This behavior is similar to that observed in Fig. 10. Thus, the reasons for trend observed in Fig. 12 are the same as explained for Fig. 10. The effect of variation of C_r on $Q^*_{h,c}$ for NTU = 2, 5 and 10 is also presented in Fig. 11.

3.1. Size limitations

The above developed theory has certain limitations in terms of channel diameter. It has already been mentioned that these equations may be used only when $Kn < 0.001$. Under this condition slip flow and rarefaction effects are negligible. This restriction on Knudsen number dictates a lower limit of microchannel diameter for which the theories based on continuum mechanics can be applied. For example, the mean free path of air is 68 nm, at STP, thereby limiting the applicability of this theory only for air flow in channels with hydraulic diameter greater than 68 μm [22]. Hong et al. have taken 60 μm as the limit below which rarefaction effects are significant when air is used as the fluid [23]. A general lower limit on the hydraulic diameter of the microchannel cannot be specified as it depends on the mean free path of gas used. For liquid flow in microchannels it may not be appropriate to use Knudsen number for determining the onset of slip boundary condition. However, for liquids slip flow condition has been experimentally proven to be nonexistent in microchannels with heights as small as 5 μm and width between 500 and 1000 μm [24].

4. Conclusion

A thermal model of a parallel flow microchannel heat exchanger subjected to external heat transfer was developed in this paper. This model can be used to predict the axial temperature as well as the effectiveness of the fluids in parallel flow microchannel heat exchangers, operating in the laminar flow regime, subjected to external heat flux. The model is limited to microchannel flow applications in which the working fluids are incompressible, single phase, maintain no-slip wall conditions, and do not exhibit any rarefaction effects ($Kn < 0.001$). In the presence of external heating the effectiveness of hot fluid decreased while that of the cold fluid increased for a specific NTU. On the other hand when subjected to external cooling the effectiveness of the hot and cold fluid improved and degraded, respectively. For unbalanced flow conditions the effectiveness of the fluids, for a particular NTU and heating level, is greatest when the hot fluid in the heat exchanger has the lowest heat capacity. Temperature cross over which is nonexistent in a parallel flow microchannel heat exchanger free of external heating may occur in the presence of external heat transfer. Temperature cross over can occur in a balanced parallel flow microchannel heat exchanger when the heat supplied to the cold fluid is greater than that supplied to the hot fluid. Temperature cross over may also occur in unbalanced parallel flow heat exchangers in which the cold fluid has the lowest heat capacity and is subjected to external heating.

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